The active role of topology: pattern formation on directed networks

MALBOR ASLLANI

DEPARTMENT OF MATHEMATICS AND NAMUR CENTER FOR COMPLEX SYSTEMS – NAXYS, UNIVERSITY OF NAMUR REMPART DE LA VIERGE 8, 5000 NAMUR, BELGIUM

Dynamical processes on networks have generated widespread interest in recent years. The theory of pattern formation in reaction-diffusion systems defined on symmetric networks has often been investigated, due to its applications in a wide range of disciplines. Here we extend the theory to the case of directed networks, which are found in a number of different fields, such as neuroscience, computer networks and traffic systems. Owing to the structure of the network Laplacian, the dispersion relation has both real and imaginary parts, at variance with the case for a symmetric, undirected network. The homogeneous fixed point can become unstable due to the topology of the network, resulting in a new class of instabilities, which cannot be induced on undirected graphs. Results from a linear stability analysis allow the instability region to be analytically traced. Numerical simulations show traveling waves, or quasi-stationary patterns, depending on the characteristics of the underlying graph.

References

1. The chemical basis of morphogenesis, A. Turing, Philosophical Transactions of the Royal Society, 327, (37–72), 1952
2. Turing patterns in network-organized activator-inhibitor systems, H. Nakao, A.S. Mikhailov, Nature Physics, 6, (544–550), 2010
3. The theory of pattern formation on directed networks, M. Asllani, J. D. Challenger, F. S. Pavone, L. Sacconi, D. Fanelli, Nature Communications, 5, (4517), 2014.